

# MANEUVERING THE MIDDLE

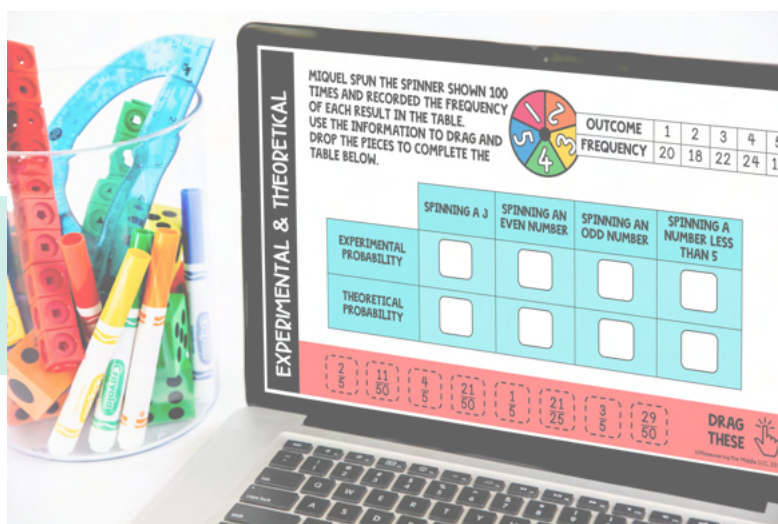
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# MANEUVERING THE MIDDLE

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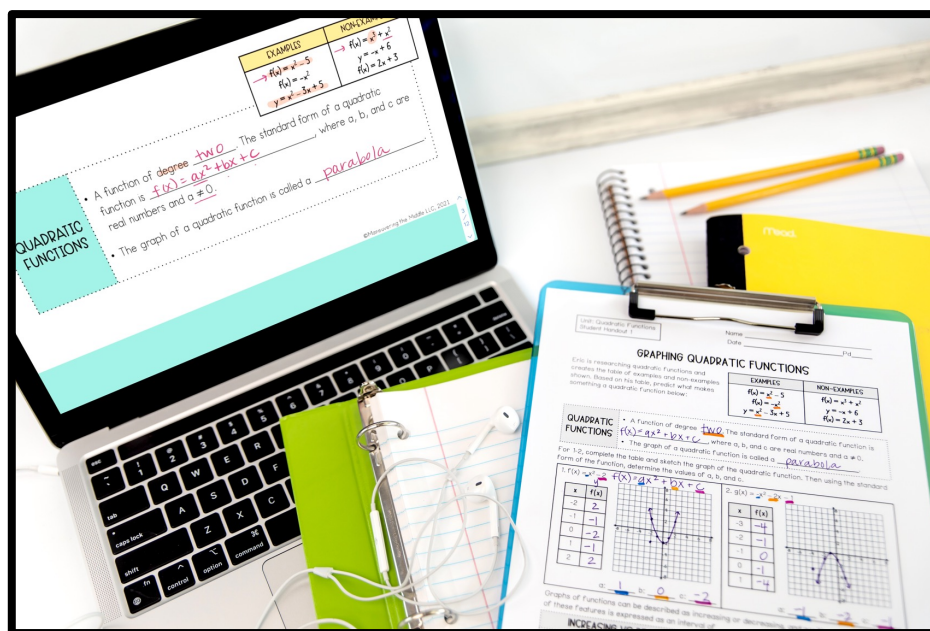
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## all access

standards-based math curriculum for grades 6-algebra 1



Math curriculum designed to meet students' needs and empower teachers.

- ✓ Grade Level Curriculum: student-friendly guided notes, hands-on activities, teaching slides (coming August 2023), and teacher planning resources
- ✓ Supplemental Digital Components: digital activities, teaching slides, Google Form™ assessments
- ✓ Student Video Library: professional quality videos aligned to student handouts

## 8<sup>th</sup> ccss planning guide

A MANEUVERING THE MIDDLE® RESOURCE

### what is it?

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This resource has been designed to model the process presented in the math training, “A Step-by-Step Plan for Unfinished Learning”. Please use the information provided to jump start your planning for the school year.

### how does it work?

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A planning guide has been included for each of the key topics in Unit 1: Real Number System. Each guide will help you prepare for formative assessment opportunities, common student misconceptions, instructional strategies you can use to reach your students and suggestions for utilizing activities to best support your students’ needs.

PAGE	TOPIC	RESOURCE
5	Ordering Real Numbers	Planning Guide
7	Classifying Real Numbers	Planning Guide
9	Estimating Square Roots	Maze Activity
14	Classifying Real Numbers	Guess My Number Activity

### learn more about All Access

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The hands-on activities included are a brief sample of one element of our All Access membership. All Access is math curriculum designed to meet students’ needs and empower teachers. You can find out more by clicking the link below.

[maneuveringthemiddle.com/math-curriculum](https://maneuveringthemiddle.com/math-curriculum)

# ordering real numbers

## goal

Students should be able to order a set of real numbers from mathematical and real-world situations.

## prior skills

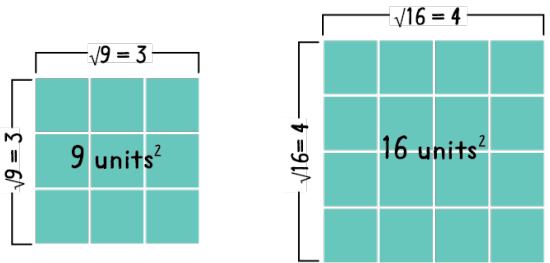
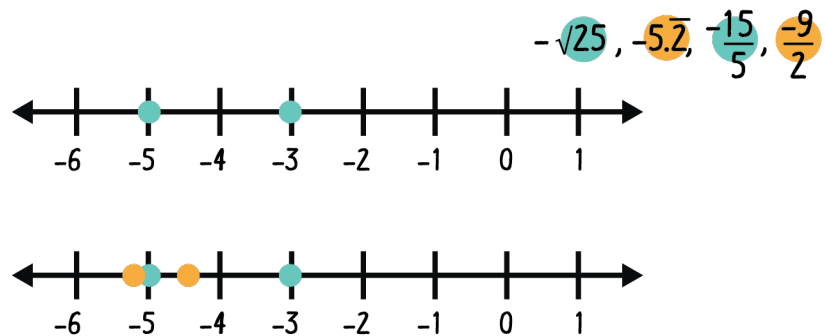
In previous grades, students converted rational numbers to different forms.

## related materials

- Unit 1, Student Handouts 1-3
- Unit 1, Student Handout 6

	estimating square roots	ordering a set of values								
formative assessments	<div>Estimate the value of <math>\sqrt{50}</math>.</div> <div>student handout 3</div> <div><math>-\sqrt{12}</math> is between ____ and ____.</div> <div>student handout 3</div>	<div>List the following in descending order.</div> <div><math>\sqrt{121}</math>, <math>\pi^2</math>, 11.1, <math>\sqrt{130}</math></div> <div>student handout 6</div> <div>List the following in increasing order.</div> <div>0.65, 6.5%, <math>\frac{2}{3}</math>, <math>\frac{1}{2}</math></div> <div>student handout 6</div>								
common misconceptions	<p>Students may divide by 2 instead of taking the square root (ex. 1) or may struggle to estimate the value of an irrational square root to the nearest tenth (ex. 2).</p> <table><tr><th>example 1</th><th>example 2</th></tr><tr><td><math>\sqrt{4} = 2</math> <math>\sqrt{16} = 8</math> <math>\sqrt{50} = 25</math></td><td><math>\sqrt{27}</math> must be between 5 and 6, so about 5.5.</td></tr></table>	example 1	example 2	$\sqrt{4} = 2$ $\sqrt{16} = 8$ $\sqrt{50} = 25$	$\sqrt{27}$ must be between 5 and 6, so about 5.5.	<p>Students may confuse vocabulary such as ascending and descending (ex. 1) or may struggle to order integers correctly (ex. 2).</p> <table><tr><th>example 1</th><th>example 2</th></tr><tr><td><math>\sqrt{121}</math>, <math>\pi^2</math>, 11.1, <math>\sqrt{130}</math> <i>If ordering the list above in descending order, a student may list the values from least to greatest instead of greatest to least.</i></td><td><math>-\sqrt{144}</math>, <math>-\frac{13}{2}</math>, -8.5, <math>-3^2</math> <i>If ordering the list above from least to greatest, a student might start with <math>-\frac{13}{2}</math>, the “smallest” negative rather than <math>-\sqrt{144}</math>, the value that is the furthest from zero.</i></td></tr></table>	example 1	example 2	$\sqrt{121}$ , $\pi^2$ , 11.1, $\sqrt{130}$ <i>If ordering the list above in descending order, a student may list the values from least to greatest instead of greatest to least.</i>	$-\sqrt{144}$ , $-\frac{13}{2}$ , -8.5, $-3^2$ <i>If ordering the list above from least to greatest, a student might start with <math>-\frac{13}{2}</math>, the “smallest” negative rather than <math>-\sqrt{144}</math>, the value that is the furthest from zero.</i>
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# ordering real numbers

	estimating square roots	ordering a set of values
instructional strategies	<p><b>model it:</b> have students cut out squares of various dimensions from graph paper to observe the relationship between the area of a square and the side length of the square; this is also a good way to reinforce the concept of “perfect squares”</p>  <p><b>model it:</b> encourage students to sketch a number line to organize their thinking and identify two integers that a non-perfect square must lie between</p>	<p><b>model it:</b> encourage students to always sketch a number line to organize their thinking and avoid errors, especially when ordering integers</p> <p><b>organize it:</b> when given a list of values to order, have students first place the whole numbers and integers on the number line (including fractions and/or square roots that simplify to integers) and then place the fractions, decimals and/or non-perfect square roots</p> 
extra practice and resources	<p><b>Estimating Square Roots Mazes*</b></p> <p>Consider using the maze that best needs the meets of your students according to the types of values included on each maze:</p> <ul style="list-style-type: none"> <li>Maze #1: Square roots of numbers <math>\leq 100</math></li> <li>Maze #2: Square roots of numbers <math>\leq 225</math></li> </ul>	<p>For extra practice, give groups of students index cards with a value written on each and ask the group to order their cards in ascending or descending order. Be sure to include different forms such as fractions, decimals, percentages, and square roots. Students could also use masking tape to create a number line and order the cards on the line.</p>

\*Estimating Square Roots Mazes is included in this PDF on pages 9-13.

# classifying real numbers

## goal

Students should be able to describe relationships between sets of real numbers.

## prior skills

In previous grades, students classified rational numbers.

## related materials

- Unit 1, Student Handout 4
- Unit 1, Student Handout 5

### distinguishing rational vs irrational numbers

### representing relationships between real numbers

#### formative assessments

Classify each given value as rational or irrational. Explain your choice in the last column.

VALUE	RATIONAL OR IRRATIONAL?	EXPLANATION
-12		
$\pi$		

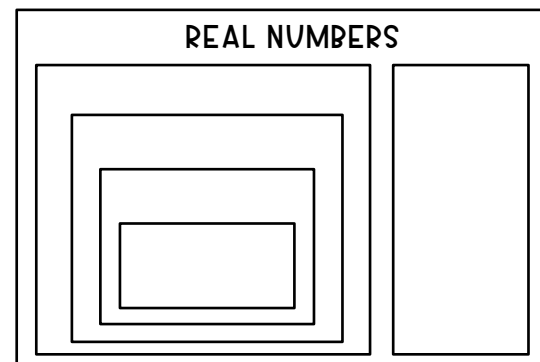
student handout 4

Place the headings for each type of real number in the graphic organizer below. Then, write the following values in the box where they belong.

VALUES:

$2\sqrt{64}$	-15	$\frac{16}{4}$
$\sqrt{75}$	0	-12.2

student handout 5



#### common misconceptions

Students may classify all square roots as irrational (ex. 1) or may forget to simplify a value before classifying (ex. 2).

example 1

$\sqrt{64}$  is an irrational value because it is represented as a square root.

example 2

The most specific set  $-\frac{36}{12}$  belongs to is rational since it is expressed as a ratio.

Students may think that numbers only belong to one set (ex. 1) or misapply logic related to sets and subsets (ex. 2).

example 1

$-\frac{15}{3} \rightarrow \text{integer}$

example 2

If all rational numbers are real numbers, then all real numbers must be rational numbers.

# classifying real numbers

	distinguishing rational vs irrational numbers	representing relationships between real numbers
instructional strategies	<p><b>rephrase it:</b> give students an opportunity to complete the following sentences in their own words:</p> <ul style="list-style-type: none"> <li>Rational values include _____</li> <li>Irrational values include _____</li> </ul> <p><b>model it:</b> allow students to input a non-perfect square root on their calculator to see how an irrational value is displayed</p>	<p><b>model it:</b> allow students to practice organizing relationships between a category they are familiar with before applying the practice to real numbers in order to solidify concepts of sets and subsets (ex. a graphic organizer that organizes the people at your school)</p> <div data-bbox="1264 436 1793 660" data-label="Diagram"> <pre> graph TD     JMS[JOHNSON MIDDLE SCHOOL]     S[STUDENTS]     T[TEACHERS]     JMS --- S     JMS --- T     S --- G[8TH GRADERS]     </pre> </div> <p>Consider asking, "Is it possible to be included in the section for 8<sup>th</sup> graders but not in the section for students? What about the other way around?"</p>
extra practice and resources	<p><b>Rational vs Irrational Numbers Card Sort*</b></p> <p>The cards include values represented as fractions, decimals and square roots. Consider having students verbally complete the sentences below when sorting a card based on the representation:</p> <ul style="list-style-type: none"> <li>This <b>fraction</b> is classified as _____ because _____</li> <li>This <b>decimal</b> is classified as _____ because _____</li> <li>This <b>square root</b> is classified as _____ because _____</li> </ul>	<p><b>Classifying Real Numbers Guess My Number**</b></p> <p>Utilize this activity in groups to reinforce the definitions and distinguishing characteristics of different types of real numbers. Consider the following ideas during the activity:</p> <ul style="list-style-type: none"> <li>Allow students to use a graphic organizer as they read each statement to show their thinking and provide a visual understanding of the clues</li> <li>Consider extending the activity by asking students to create their own clues to describe a mystery number</li> </ul>

\*Rational vs Irrational Numbers Card Sort is included as a part of the 8<sup>th</sup> Grade All Access Membership.

\*\*Classifying Real Numbers Guess My Number is included in this PDF on pages 14-21.



# ESTIMATING SQUARE ROOTS

## MAZE ACTIVITY

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Students will be able to approximate and locate irrational numbers on a number line.



**8.NS.2** Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g.,  $\pi^2$ ).



**8.2B** Approximate the value of an irrational number, including  $\pi$  and square roots of numbers less than 225, and locate that rational number approximation on a number line

**Ideas for Implementation:** This activity challenges student ability to estimate non-perfect square roots. There are two different mazes included. Maze #1 includes square roots of numbers less than or equal to 100 and Maze #2 includes square roots of numbers less than or equal to 225.

### Instructions:

- 1) Print and copy a maze (or both mazes) for each student.
- 2) Students will pick the expression that results in the best estimate of the point plotted on each number line. Accurate selections will lead them correctly to the finish.

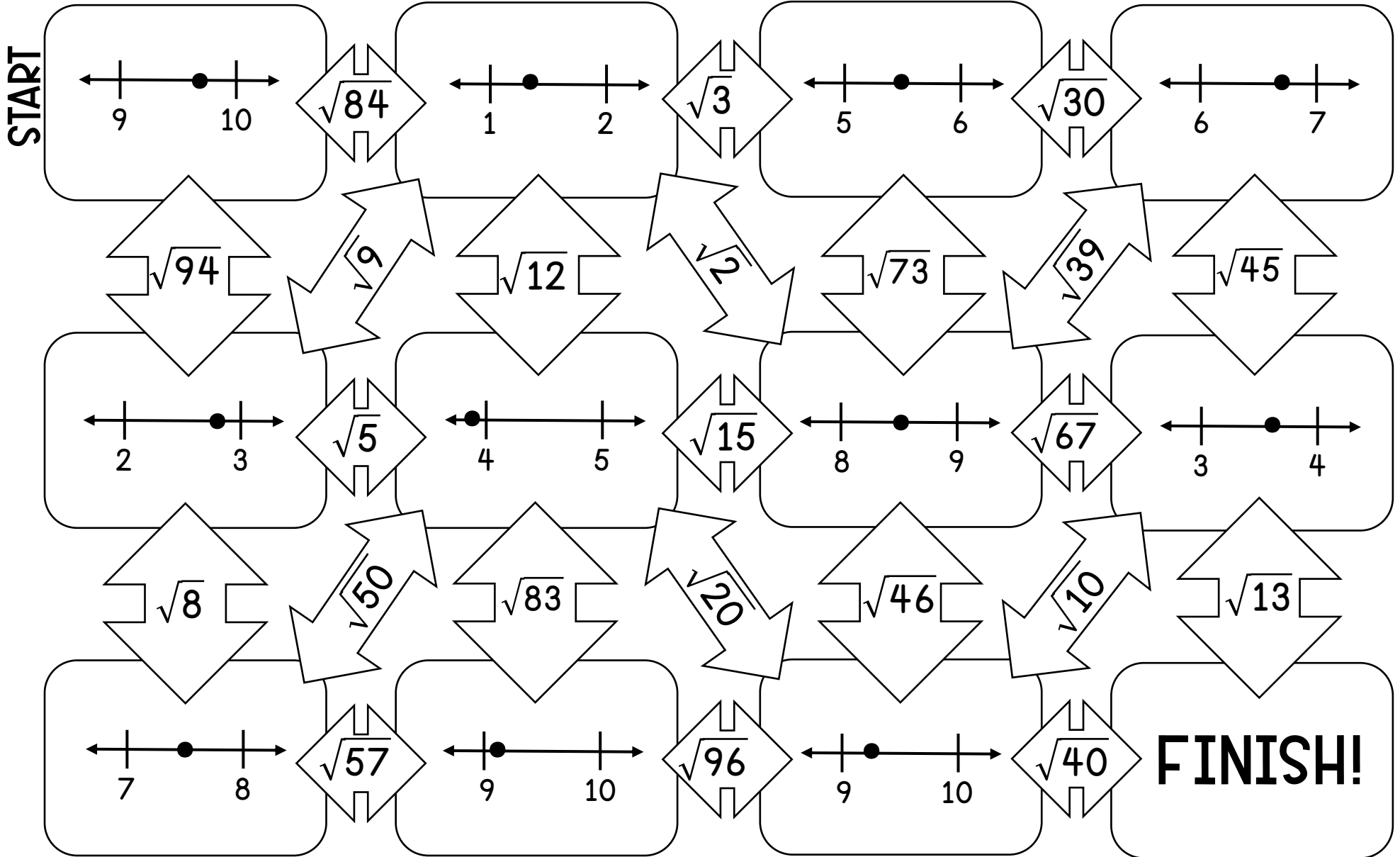
**Notes:** Consider having students shade the path with a highlighter or colored pencils for easy grading. This activity is best used as an individual assessment and is great for early finishers.

# ESTIMATING SQUARE ROOTS

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Pd: \_\_\_\_\_

# MAZE #1

**Instructions:** Follow the best estimate of each square root plotted on the number lines to make it correctly through the maze. Shade or color your path as you go.

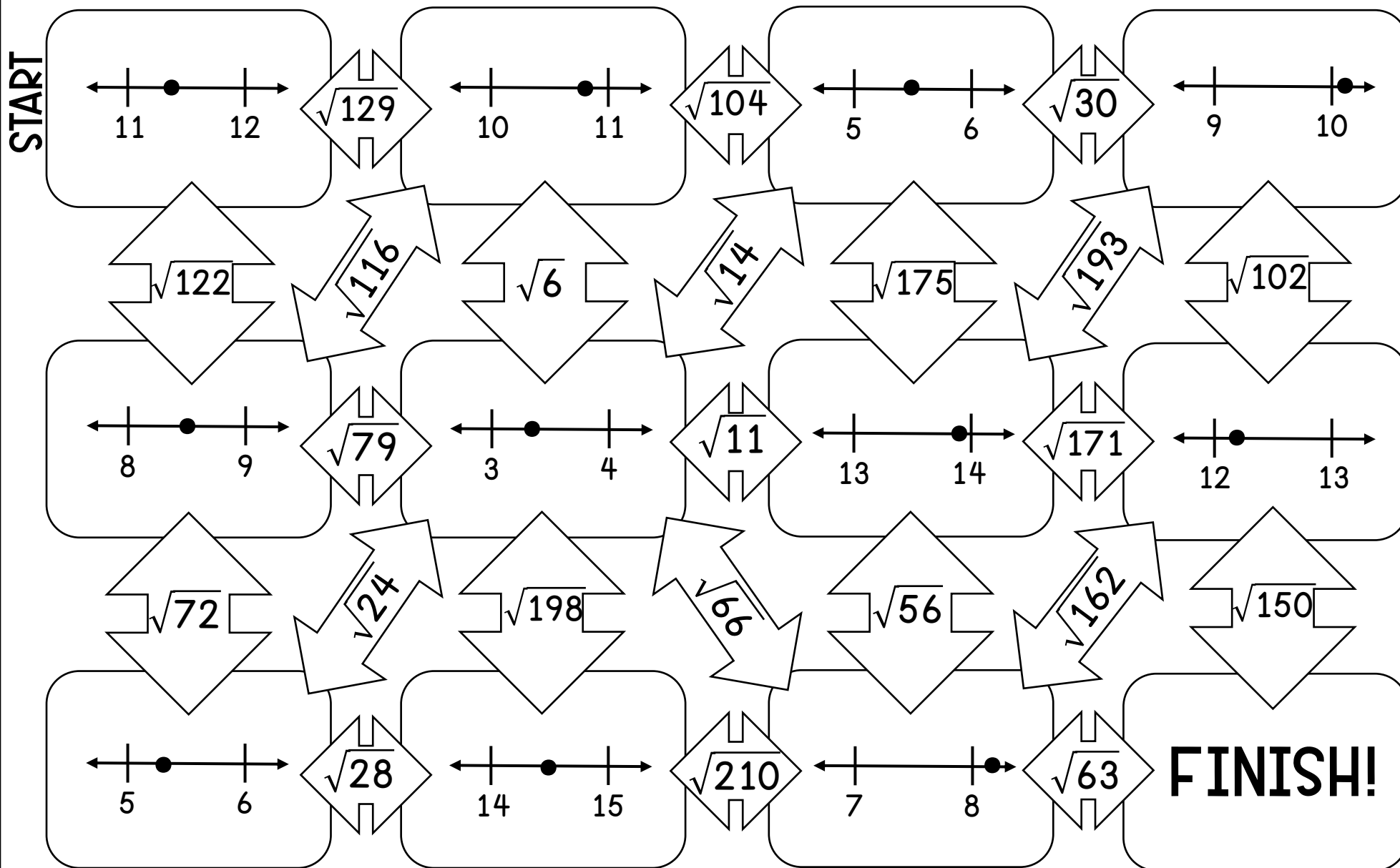


# ESTIMATING SQUARE ROOTS

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Pd: \_\_\_\_\_

## MAZE #2

Instructions: Follow the best estimate of each square root plotted on the number lines to make it correctly through the maze. Shade or color your path as you go.

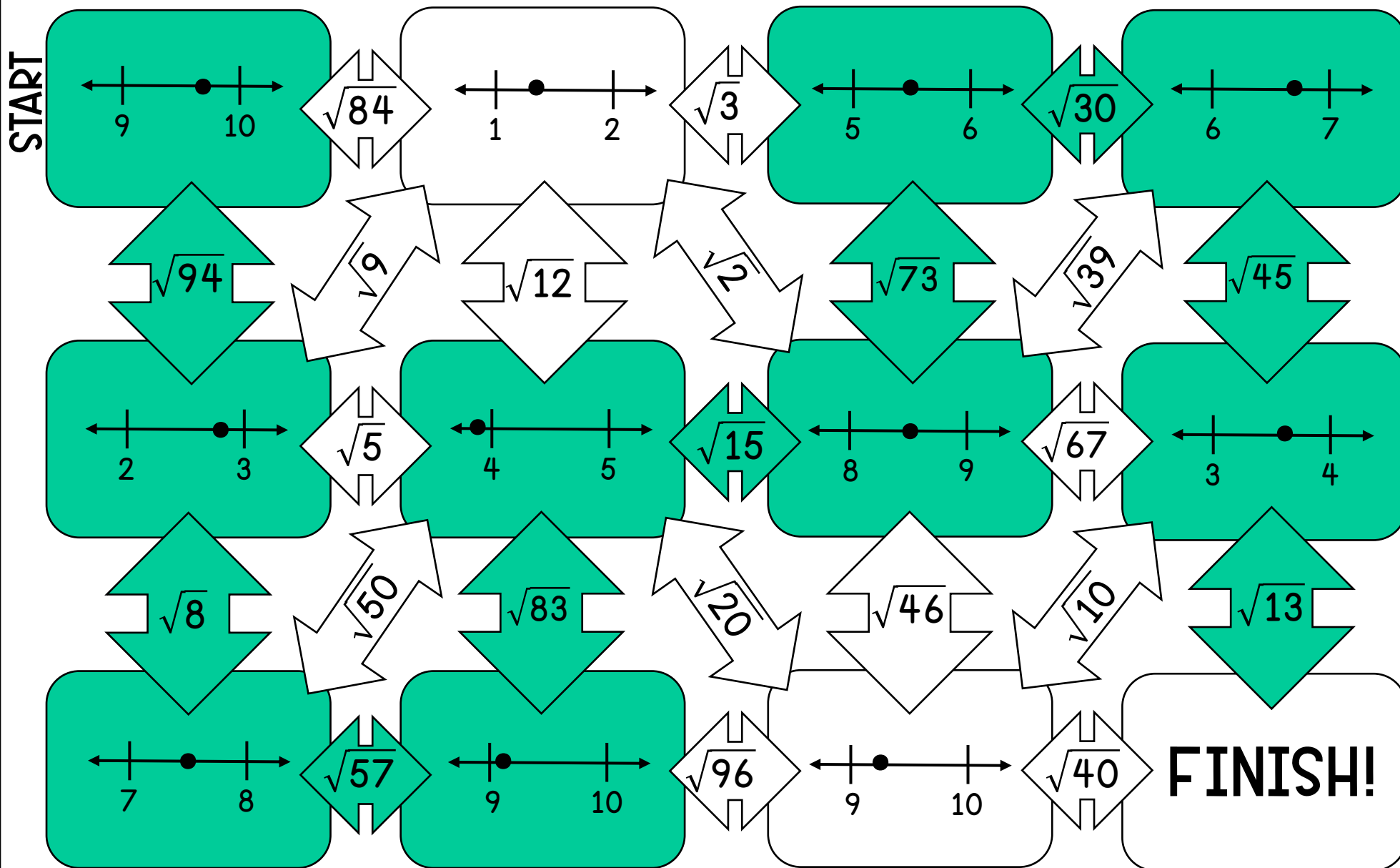


# ESTIMATING SQUARE ROOTS

Name: KEY Date: \_\_\_\_\_ Pd: \_\_\_\_\_

## MAZE #1

Instructions: Follow the best estimate of each square root plotted on the number lines to make it correctly through the maze. Shade or color your path as you go.

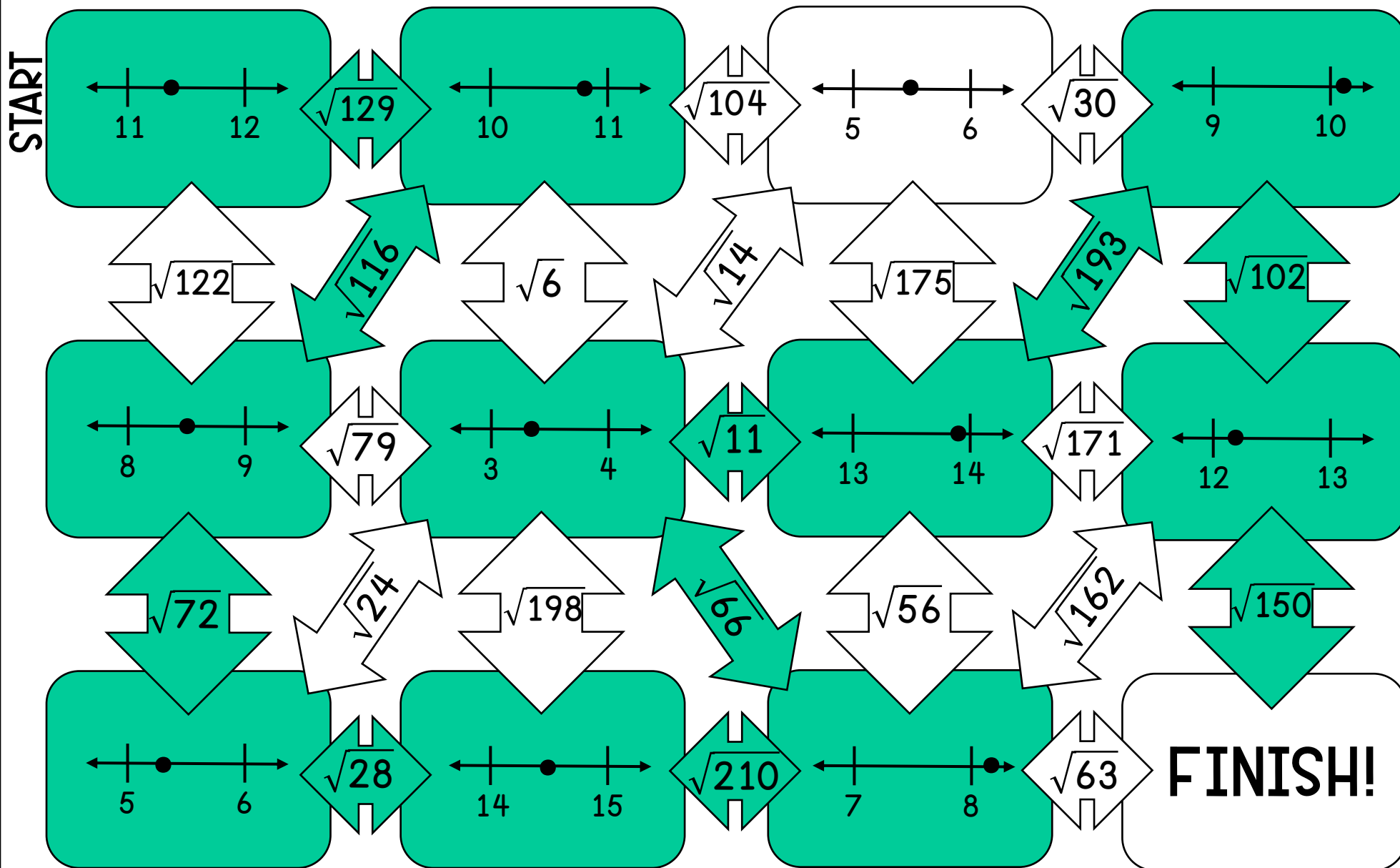


# ESTIMATING SQUARE ROOTS

Name: KEY Date: \_\_\_\_\_ Pd: \_\_\_\_\_

## MAZE #2

Instructions: Follow the best estimate of each square root plotted on the number lines to make it correctly through the maze. Shade or color your path as you go.



# CLASSIFYING REAL NUMBERS

## *"Guess My Number"*

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Students will be able to classify types of real numbers.



**8.NS.1** Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.



**8.2.A** Extend previous knowledge of sets and subsets using a visual representation to describe relationships between sets of real numbers.

**Ideas for Implementation:** This activity reinforces student's knowledge of real numbers and sets of real numbers as students use clues to correctly identify a number being described.

### **Instructions:**











- 1) Print and copy a recording sheet for each student.
- 2) Print and copy the clue cards onto cardstock and cut into half sheets.
- 3) Students can start at any clue card and will read the clues to correctly match all 10 numbers to their clues.

**Notes:** You could make one set of the clues and post them around the room for students to walk to in small groups. Otherwise, cards could be placed on a binder ring in order to give individual sets of clue cards to smaller groups.

## GUESS MY NUMBER: RECORDING SHEET

**INSTRUCTIONS:** The 10 cards with clues describe the 10 numbers in the table below. Use the clues given on the 10 cards and your knowledge of real numbers in order to help you correctly match each number. Record your guesses by each magnifying glass.

-10	$\sqrt{70}$	3	$\sqrt{120}$	0
$-12\frac{2}{3}$	1.8	$-\sqrt{225}$	$\frac{4}{3}$	$\pi$

 _____	 _____
 _____	 _____
 _____	 _____
 _____	 _____
 _____	 _____

- I AM NOT IRRATIONAL
- I AM AN INTEGER
- I AM A NATURAL NUMBER
- I AM A PRIME NUMBER
- WHAT NUMBER AM I?



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- I AM IN A SUBSET OF RATIONAL NUMBERS
- I AM NOT NEGATIVE
- I AM A WHOLE NUMBER
- I AM NOT A NATURAL NUMBER
- WHAT NUMBER AM I?



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- I AM NOT IRRATIONAL
- MY OPPOSITE IS A NATURAL NUMBER
- I AM NOT A WHOLE NUMBER
- I AM GREATER THAN -13
- WHAT NUMBER AM I?



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- I AM A REAL NUMBER
- I AM A RATIONAL NUMBER
- I AM NOT A NATURAL NUMBER
- I AM LESS THAN -13
- WHAT NUMBER AM I?



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- I AM NOT IRRATIONAL
- I AM NOT A WHOLE NUMBER
- I AM NOT AN INTEGER
- I AM NOT POSITIVE
- WHAT NUMBER AM I?



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- I AM A REAL NUMBER
- I AM NON-TERMINATING AS A DECIMAL
- I AM NOT A WHOLE NUMBER
- I AM BETWEEN 8 & 9
- WHAT NUMBER AM I?



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- I AM NOT AN INTEGER
- I AM NOT A REPEATING DECIMAL
- I AM IMPOSSIBLE TO WRITE AS A FRACTION
- I AM BETWEEN 3 & 4
- WHAT NUMBER AM I?



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- I CAN BE WRITTEN AS A FRACTION
- I AM NOT A NATURAL NUMBER
- I AM NOT AN INTEGER
- I AM A TERMINATING DECIMAL
- WHAT NUMBER AM I?



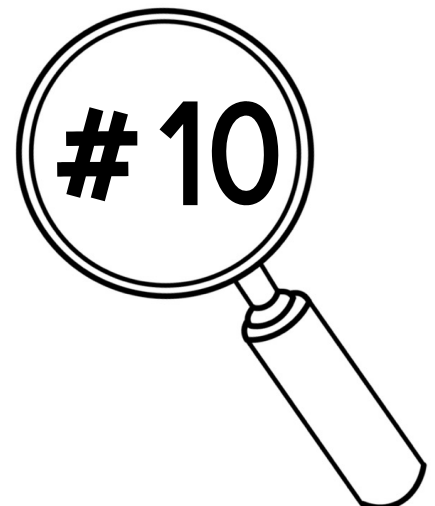
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- I CAN BE WRITTEN AS A RATIO
- I AM NOT A NEGATIVE NUMBER
- I AM NOT A WHOLE NUMBER
- I WOULD BE REPEATING AS  
A DECIMAL
- WHAT NUMBER AM I?



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- I AM NOT A NATURAL NUMBER
- I AM NOT A TERMINATING DECIMAL
- I CAN'T BE WRITTEN AS A RATIO
- I AM GREATER THAN 10 BUT  
LESS THAN 11
- WHAT NUMBER AM I?













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## GUESS MY NUMBER: RECORDING SHEET

**INSTRUCTIONS:** The 10 cards with clues describe the 10 numbers in the table below. Use the clues given on the 10 cards and your knowledge of real numbers in order to help you correctly match each number. Record your guesses by each magnifying glass.

-10	$\sqrt{70}$	3	$\sqrt{120}$	0
$-12\frac{2}{3}$	1.8	$-\sqrt{225}$	$\frac{4}{3}$	$\pi$

 <div>#1</div> <div>3</div>	 <div>#6</div> <div><math>\sqrt{70}</math></div>
 <div>#2</div> <div>0</div>	 <div>#7</div> <div><math>\pi</math></div>
 <div>#3</div> <div>-10</div>	 <div>#8</div> <div>1.8</div>
 <div>#4</div> <div><math>-\sqrt{225}</math></div>	 <div>#9</div> <div><math>\frac{4}{3}</math></div>
 <div>#5</div> <div><math>-12\frac{2}{3}</math></div>	 <div>#10</div> <div><math>\sqrt{120}</math></div>